

Delta-Sigma and 24-bit Recording

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This tutorial describes the principles of 24-bit recording systems and clarifies some common mis-conceptions regarding these systems.

This is a general treatment of the subject and applies to I/O System Two, Sercel 388, Aram24, Opseis Eagle and any other seismic recording systems employing current delta-sigma technology.

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Delta-Sigma and 24-bit Recording

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Moving Through This Tutorial

- Pressing the **PGDN** key or the **ENTER** key will advance to the next slide
- Pressing the **PGUP** key or the **BKSP** key will reverse to the previous slide
- If you wish to jump to a specific **Slide Number** (located in lower right corner of each slide) type the number and press **ENTER**

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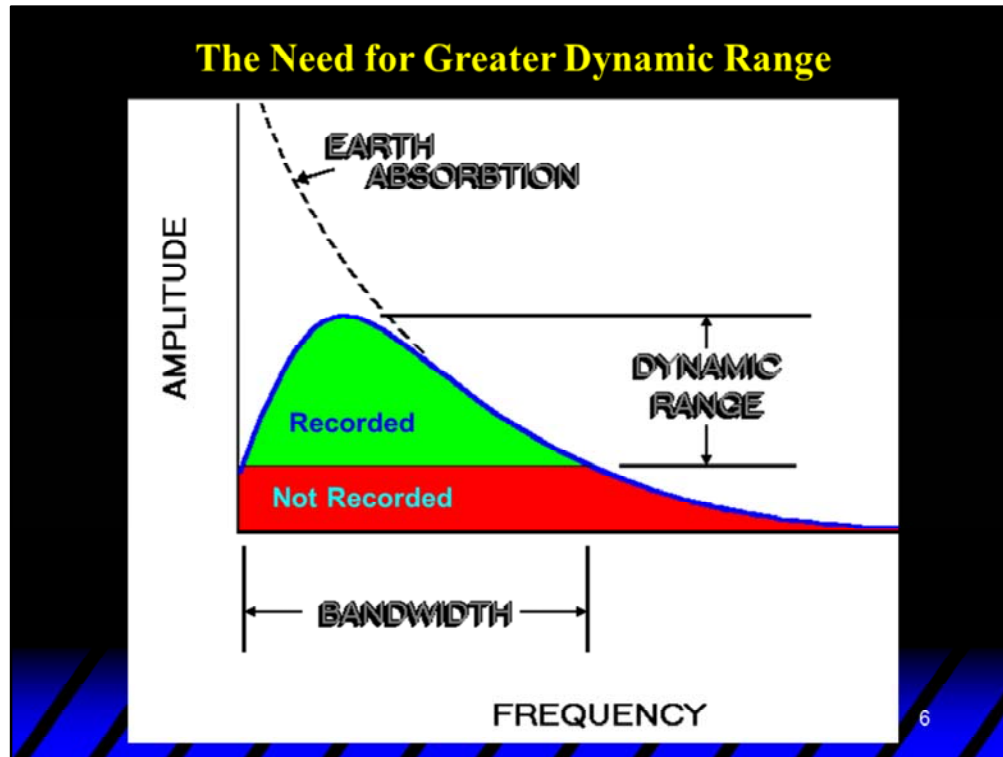
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The Need for Delta-Sigma and 24-bit recording

- Recoverable bandwidth in seismic records is limited by the combination of earth's absorption of higher frequencies and the limited dynamic range of our recording and processing systems.
- We must re-shape the incoming spectrum ...
- Or increase instantaneous dynamic range.

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It is well established in the seismic industry that optimum image quality and interpretive uniqueness are achieved by maximizing the bandwidth where signal prevails over noise. Our objective throughout design, acquisition, processing and interpretation should be to optimize recoverable bandwidth.



A typical amplitude spectrum of seismic data shows limited low frequency recovery due to limits of measurement in broadband instrumentation and limited high frequencies due to natural earth absorption.

Each cycle of wave propagation results in some frictional loss in the elastic earth. Since high frequencies experience more cycles over a given interval of material, they experience more attenuation.

Coupled with limited dynamic range of our seismic system, this results in limited recoverable bandwidth in our data.

In older IFP instruments, the most severe limit on dynamic range was imposed by the successive approximation ("ladder" style) A-D converter. Although these devices have a theoretical dynamic range of 84 dB, the practical limit in use is more like 50 to 70 dB.

Delta Sigma and Over Sampling

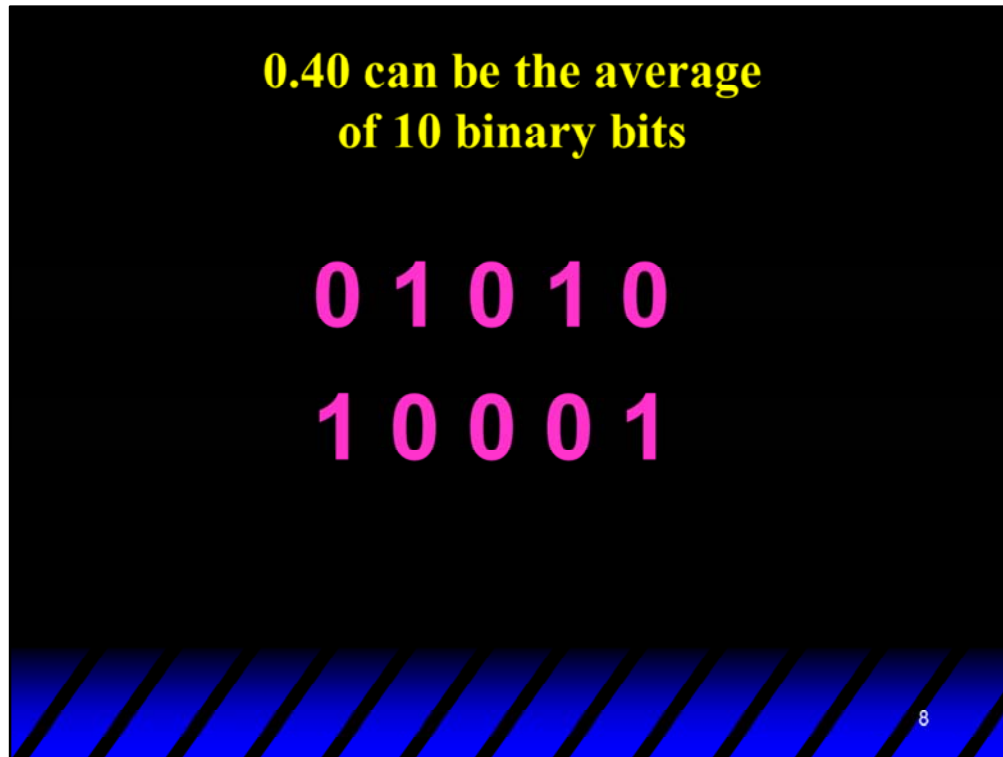
- The amplitude of any sample can be recorded as a series of binary 1's and 0's (or bits).
The value of the sample is the average of the bits.
- The more bits that we record, the greater the precision (or dynamic range) of the recorded number.

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The concept of over-sampling is to record more samples (faster sample rate) than conventional sampling theory would dictate. The Nyquist Theorem dictates that we should preserve a minimum of two samples per cycle.

For every desired output sample (usually 2 millisecond intervals), oversampling will generate many samples. We then average these samples to produce one value for each desired output sample. This averaging reduces noise.

Most of today's seismic D-S recording systems generate 512 values every 2 ms. This represents a sampling frequency of 256,000 Hz (256 KHz) or a Nyquist frequency of 128,000 Hz.



As an example, consider taking the average of these 10 binary numbers. If our input voltage was exactly 0.400 volts, these few bits could accurately represent that voltage.

An input signal of 0.50 volts would require that one of the “zeros” be changed to a “one”.

But what if we have a signal of 0.430 volts at some instant in time? These ten bits cannot accurately measure this voltage and a round-off error of 0.03 volts will occur.

**0.43 requires more precision and must
be estimated by the average
of 100 binary bits**

```
0 1 0 1 0 0 1 0 0 1 0 1 1 0 1 0 0 1 0 1
1 0 0 1 0 0 1 0 1 0 0 1 0 1 0 0 1 0 0 1
1 0 0 1 0 1 0 1 0 1 1 0 0 1 1 0 0 1 0 1
0 1 0 0 0 0 1 0 0 1 0 1 0 0 1 0 0 1 0 1
1 0 0 1 0 1 0 0 0 1 1 0 1 0 1 0 0 1 0 1
```

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We could accurately estimate our input of 0.430 volts by averaging one hundred binary bits (43 ones and 57 zeros). A signal of 0.432 volts could not be accurately measured but would result in a round-off error of only 0.002 volts.

Notice that as we increase the number of bits which define a number, we increase the precision of our measurement. Even a single bit recording device can record very precise values if there is enough time to dramatically over sample each value.

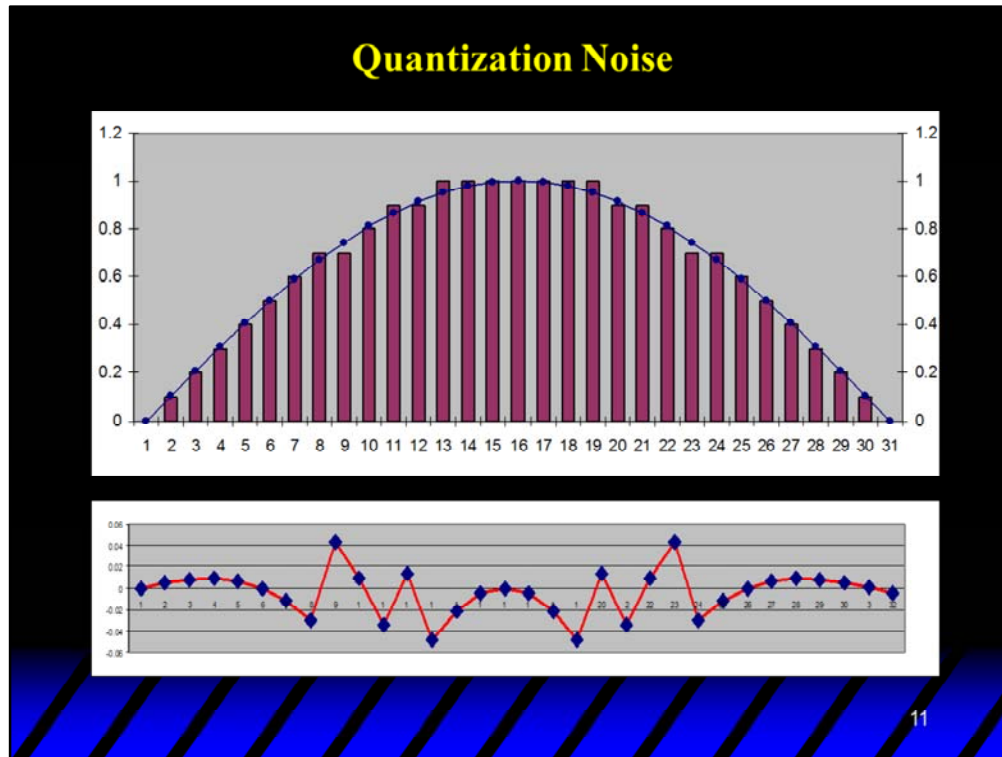
Delta Sigma and Over Sampling

- If our input voltage is slightly different, we may not be able to measure it exactly with our system (consider the last slide with an input of 0.432 volts)
- The difference between the exact input voltage and our digital measurement of it is known as **quantization noise**
- Quantization noise is quite random and therefore will distribute uniformly over all frequencies from zero to the nyquist frequency

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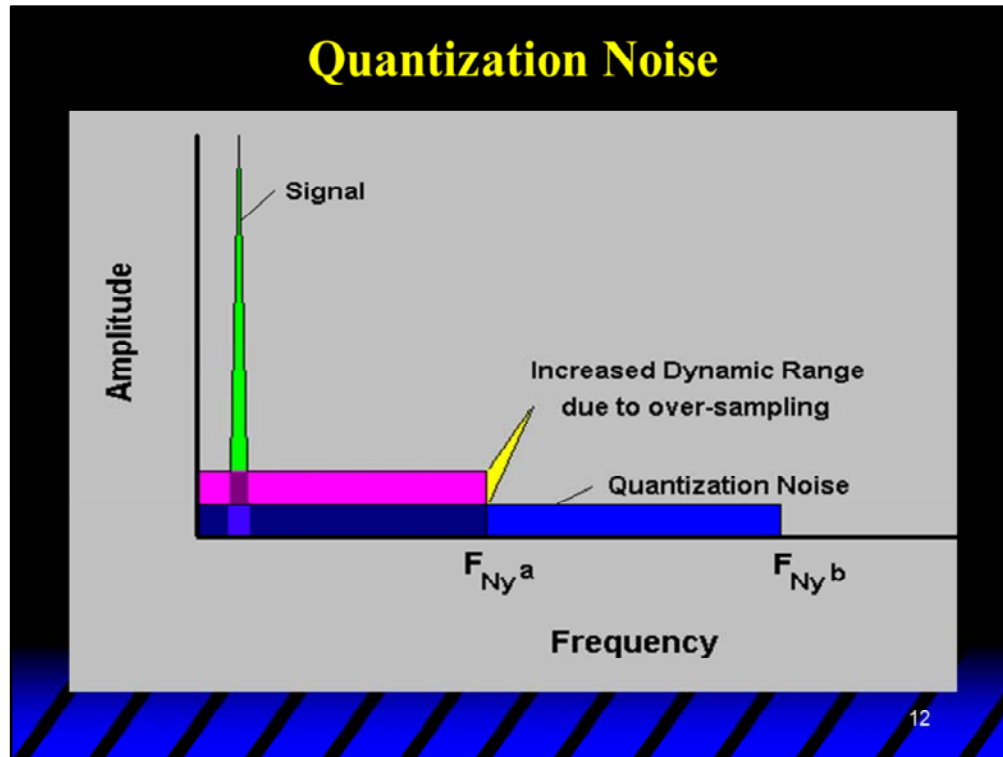
The difference between the actual magnitude of the input being measured and our estimate of that magnitude is what we have been calling “round-off error”. A more appropriate name for this inaccuracy is “Quantization Noise”.

For signals with a lot of variation (like seismic signals), the quantization error varies randomly from sample to sample. Therefore, it is regarded as white noise (evenly distributed over the frequency spectrum).



This picture shows half of a sine wave sampled precisely (in blue dots) representing an analogue signal. The purple bars represent digitized values with a certain round-off error (known as “Quantization Noise”).

The graph at the bottom shows the difference between the input signal and the limited resolution digital output. For normal seismic signals with substantial variation, this Quantization Noise tends to be random. Therefore, in the frequency domain, it will distribute itself as a low level amplitude at all frequencies from zero to the sampling nyquist frequency.



The total integral of the quantization noise over time will be a constant for different sample rates. The error at each sample will always be +/- one half of the least significant bit.

By sampling twice as fast, the nyquist frequency will be doubled. The fixed amount of quantization noise will be “spread out” over twice the frequency range. The average amplitude of the noise must then be lower at each frequency.

In this manner, over-sampling “thins” out the quantization noise and provides greater dynamic range between desired signals and measurement noise.

Delta Sigma and Over Sampling

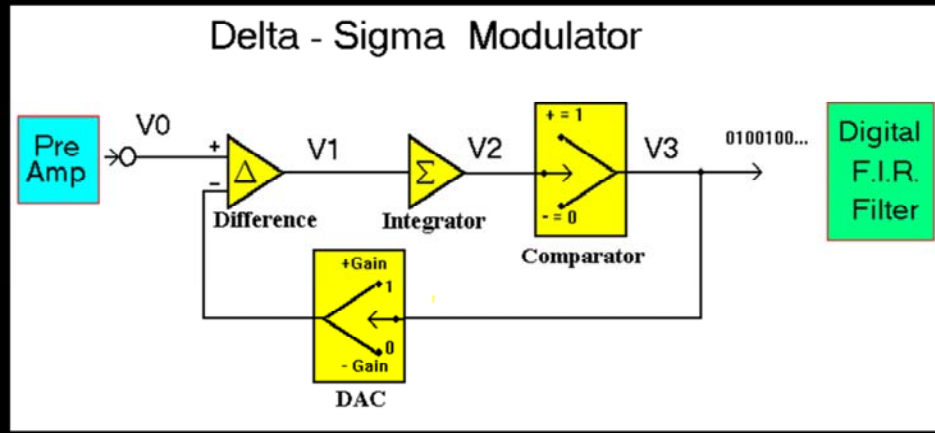
- We need a device which will rapidly generate a series of bits whose average equals the input voltage.
- Current systems work on a 256,000 Hz clock.
- Therefore, 512 bits will be generated every 2 milliseconds (512 times oversampling).
- Note that with one millisecond recording, only 256 bits are generated for each sample and we lose some precision and dynamic range.

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Successive approximation A-D converters are too slow to operate at the high sample frequencies required for significant over-sampling.

D-S modulators are a completely different type of A-D converter. They are basically a single-bit sampler that works on a 256 KHz clock. That is, they are a computer circuit which will perform a loop of operations; and 256,000 loops (or clock cycles) will be completed every second.

The Delta-Sigma Modulator



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The D-S modulator is a simple electronic loop which samples the signal 256,000 times each second. With each clock cycle, the device will produce one binary bit at its output. Every 2 milliseconds, it will produce 512 bits. These bits are averaged by the digital F.I.R. filter (finite impulse response filter).

The object of this device is to produce a stream of bits whose average equals the input voltage with a high degree of precision.

The first Op-Amp (operational amplifier) takes the difference between the input and a feedback voltage determined by the output. This difference is presented to the second Op-Amp, whose function is to accumulate this difference with the sum of all past differences (from previous clock cycles). The comparator is simple sign bit converter which will write a binary "1" if the value in the integrator is positive or a binary "0" if it is negative.

The feedback loop will present a +1.0000 voltage to the differencing Op-Amp if the output is a "1" and a -1.0000 voltage if it is negative. Since this feedback is subtracted from the input, it is negative feedback.

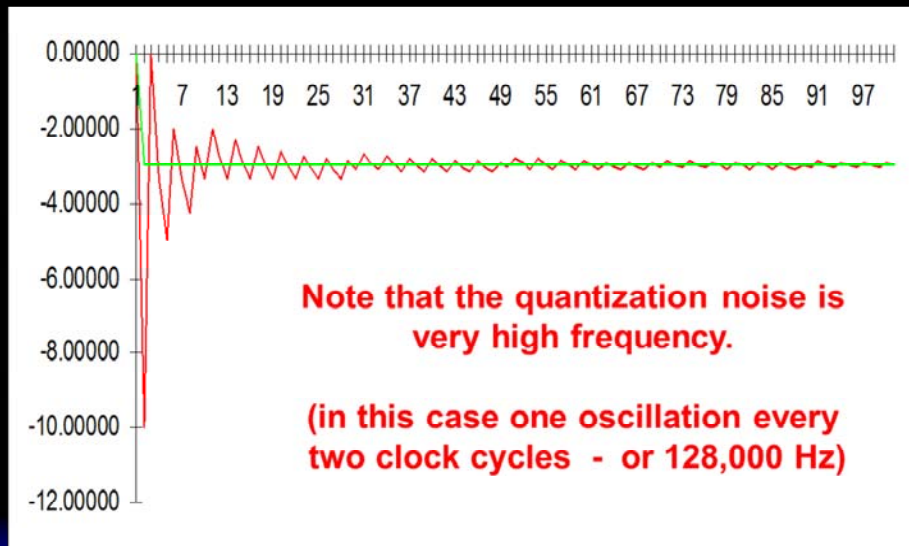
The result is that when the accumulated value in the integrator is positive (larger than the input signal), the negative feedback loop will reduce the next sample. When the integrator goes negative (output is too small compared to input), the negative feedback loop will increase the next sample.

Cycle Number	Input Voltage	Difference Voltage	Integrated Voltage	1-Bit A/D	Running Average
0	0.00000	0.00000	0.00000	0	0.00000
1	-3.00000	-3.00000	-3.00000	-10	-10.00000
2	-3.00000	7.00000	4.00000	10	0.00000
3	-3.00000	-13.00000	-9.00000	-10	-3.33333
4	-3.00000	7.00000	-2.00000	-10	-5.00000
5	-3.00000	7.00000	5.00000	10	-2.00000
6	-3.00000	-13.00000	-8.00000	-10	-3.33333
7	-3.00000	7.00000	-1.00000	-10	-4.28571
8	-3.00000	7.00000	6.00000	10	-2.50000
9	-3.00000	-13.00000	-7.00000	-10	-3.33333
10	-3.00000	7.00000	0.00000	10	-2.00000
11	-3.00000	-13.00000	-13.00000	-10	-2.72727
12	-3.00000	7.00000	-6.00000	-10	-3.33333
13	-3.00000	7.00000	1.00000	10	-2.30769
14	-3.00000	-13.00000	-12.00000	-10	-2.85714
15	-3.00000	7.00000	-5.00000	-10	-3.33333
16	-3.00000	7.00000	2.00000	10	-2.50000
17	-3.00000	-13.00000	-11.00000	-10	-2.94118
18	-3.00000	7.00000	-4.00000	-10	-3.33333
19	-3.00000	7.00000	3.00000	10	-2.63158
20	-3.00000	-13.00000	-10.00000	-10	-3.00000

This is a numerical summary of the voltages present at various points of the D-S device for the first 20 clock cycles. The running average represents the output of the FIR filter as the over sampling increases from 1 towards 512.

This output is graphed in the next picture.

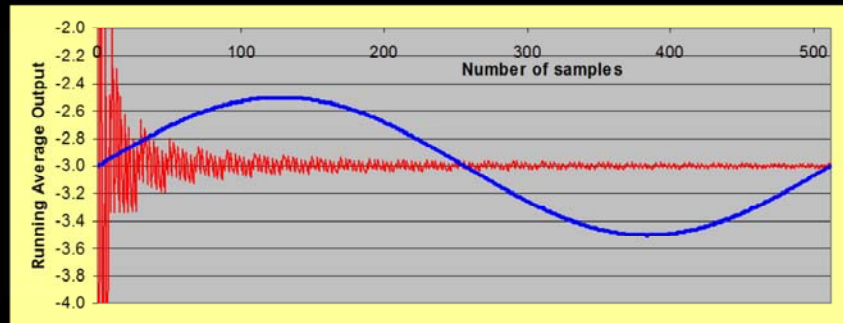
The Averaged Output for 100 clock cycles



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The green line is the steady-state input signal. The red line is the averaged output of the FIR filter. Only the first 100 samples are graphed. In a normal D-S device, there will be 512 samples every 2 ms. The oscillations of the output back and forth across the desired signal represent quantization noise. Note that these oscillations occur at a dominant frequency of about 128,000 Hz. A high cut filter of about 200 Hz would eliminate virtually all of this quantization noise and leave behind a very precise estimate of the input signal over the frequency range of 0-200 Hz (normal band of interest in reflection seismic).

The Averaged Output for 512 clock cycles



Quantization Noise
is about 128 KHz

Delta Sigma and Noise Shaping

- A single order over-sampling delta-sigma converter will shape quantization noise out of the lower frequency bands and into the higher frequency bands.
- Think of this process like squeezing a half full tube of toothpaste so that most of the toothpaste is squeezed out of one end into the opposite end.

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Rather than spreading quantization noise uniformly over the frequency range from 0 to the sampling nyquist (as depicted in figure 12), the D-S device shapes the noise so that the majority of the noise falls into the very high frequencies. It effectively removes the noise from the lower frequency range which is of prime interest to us.

Delta Sigma and Noise Shaping

- Higher order delta-sigma devices compound the noise shaping effect.
- A higher order modulator is achieved by adding another integrator.
- Recall that the frequency response of an integrator is: $\frac{1}{f}$

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A more complex device compounds this noise-shaping characteristic. In fact, for each integrator present in the modulator, we raise the power of the noise shaping function.

Delta Sigma and Noise Shaping

- The response of a first order modulator can be considered in the frequency domain as:

$$\text{Output} = (\text{Input} - \text{Output}) \times \text{Integrator Response} + Q$$

(where Q is Quantization noise of the 1-bit converter)

$$\text{or: } Y = \frac{(X - Y)}{f} + Q$$

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Simply, the output of the device can be described as the input minus the output (the result of the differencing Op-Amp), multiplied by the frequency domain response of an integrator (which behaves like a low pass filter with a response inversely proportional to frequency). The comparator estimates only the sign bit of the value in the integrator and therefore introduces some additive Quantization noise.

Delta Sigma and Noise Shaping

$$Y = \frac{(X - Y)}{f} + Q \text{ can be rearranged as}$$

$$Y = \frac{X}{f + 1} + \frac{Qf}{f + 1}$$

$$\text{at low (seismic) frequencies} \quad Y \cong X$$

$$\text{and at very high frequencies} \quad Y \cong Q$$

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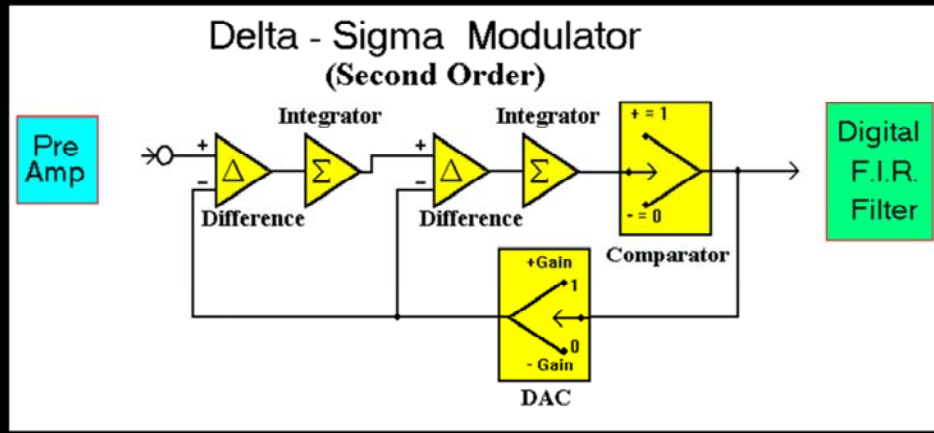
Rearranging the previous formula with all output terms on the left side of the equation gives us the form of the transfer function of this device.

We can estimate what happens at low frequencies (i.e. low relative to the sampling frequency of 256,000 Hz) and find that the output is very nearly equal to the input with no quantization noise.

We can also estimate the effect at very high frequencies and find that these are dominated by quantization noise.

Therefore, by applying a high cut filter, we can virtually eliminate quantization noise from our estimate. This is performed as an integral part of the decimation filtering in the FIR filter.

A Second Order Modulator



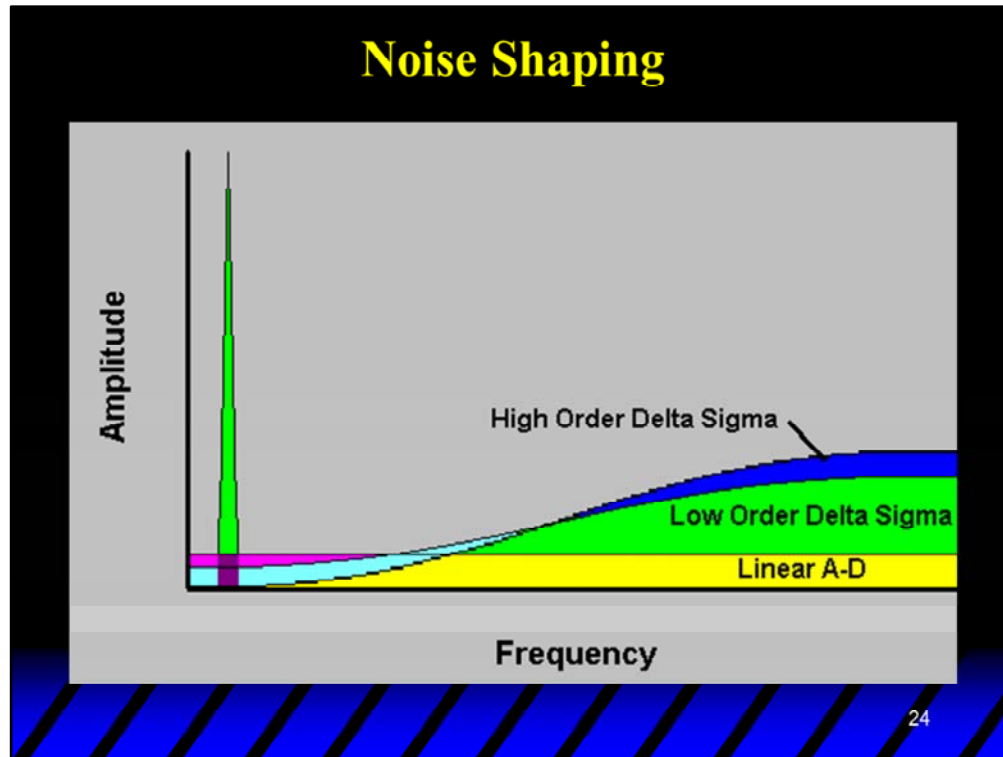
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Here is an example of a second order D-S modulator. Note that it contains two integrators and will therefore compound the noise shaping characteristic. Most seismic recording systems employ fifth order modulators.

Delta Sigma and Noise Shaping

- Each order adds one integrator to the delta-sigma modulator.
- The frequency response of the modulator will be modified to : $(1/f)^N$

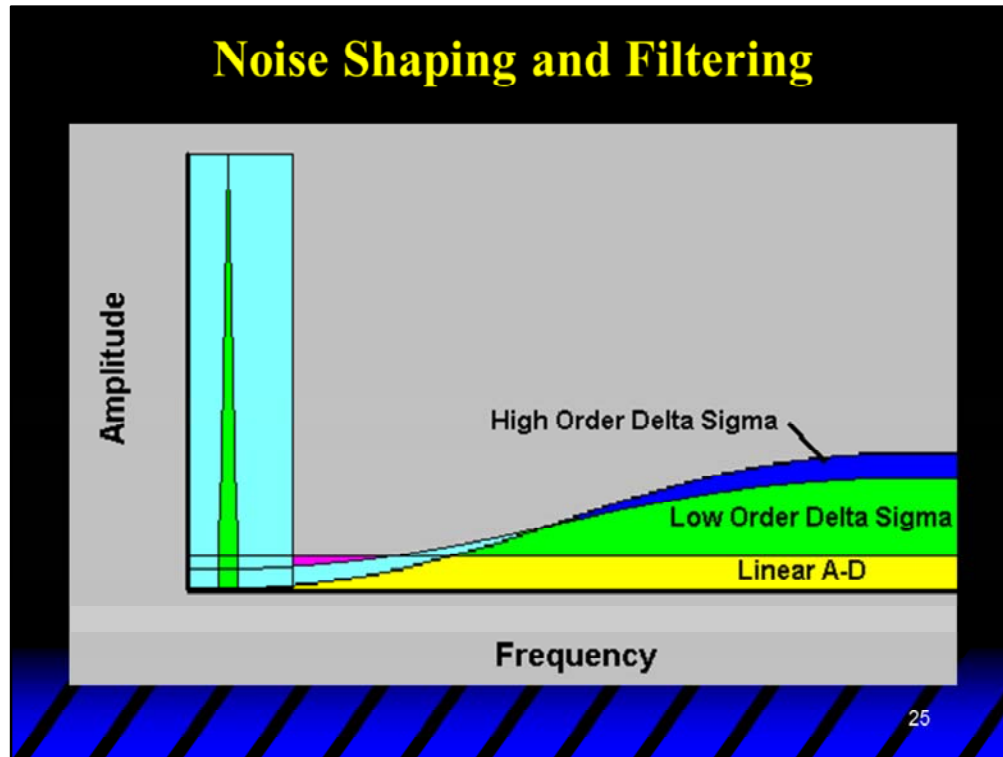
where N is the order (number of integrators)



A successive approximation A-D converter (or linear device) can be referred to as a zeroth order device because it contains no integrator. It will distribute quantization noise uniformly over the band from 0 to the sampling frequency.

Note how higher order D-S devices “squish” the noise out of our seismic bandwidths and into the very high frequencies where they can be filtered out.

The normal high cut filters for today's systems are 135 Hz (I/O and Sercel “1/2 Nyquist filter”) or 205 Hz (I/O and Sercel “3/4 Nyquist filter”) or about 160 Hz (Aram24 and Opseis).



In the FIR Filter (Finite Impulse Response) that follows the modulator, we apply a low-pass filter to keep only the lower frequencies of interest to us. The higher frequencies, that are dominated by quantization noise, are rejected.

Delta Sigma and Decimation Filtering

- We use fifth order converters in today's seismic instruments.
- The FIR (Finite Impulse Response) filter averages the many output bits and filters out the frequencies above our range of interest. The output is a highly accurate 32 bit floating point number (24 bits of mantissa and sign, 8 bits of exponent)
- The FIR filter is a high precision digital filter acting on highly sampled data.

It cannot be mimicked in subsequent processing.

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The filtering performed in the FIR filter is a high fidelity digital process conducted on data sampled at 256,000 Hz. The result is a high precision 32 bit floating point number with a 24-bit mantissa and 8 bit exponent. The instantaneous dynamic range is determined by the mantissa and these devices are referred to as 24-bit converters.

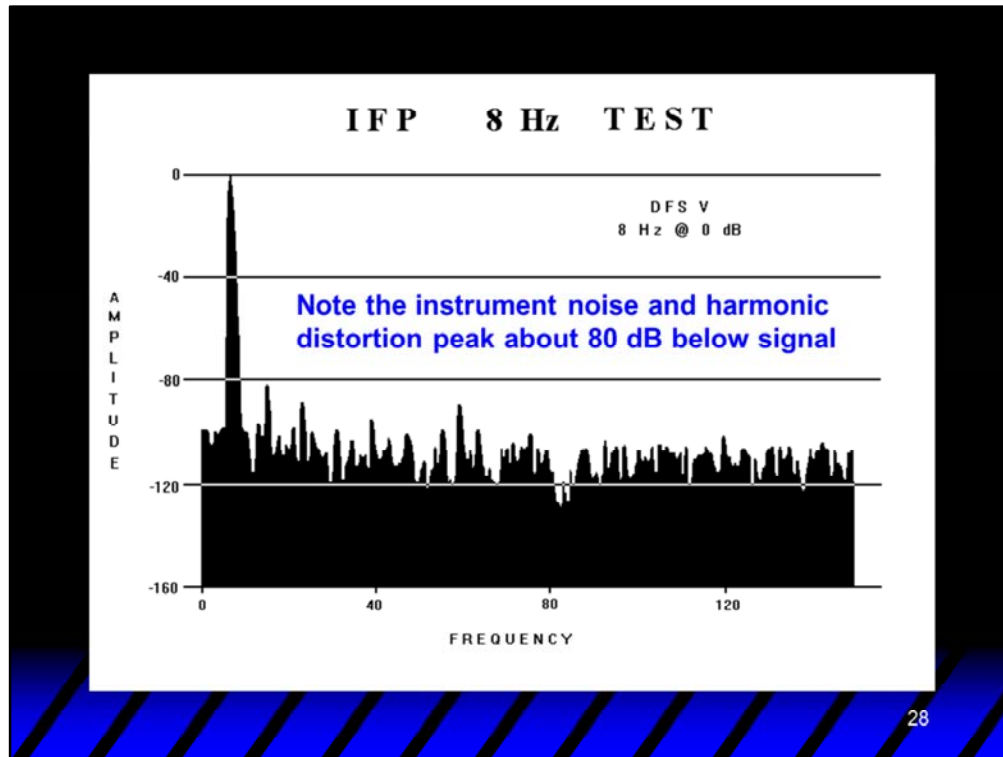
The I/O system also offers a low cut filter as an integral part of the FIR filter. Note that this is also a high fidelity filter applied to rapidly sampled data. It is not the same as applying a digital filter to the decimated data set in later stages of processing. We have found some situations where careful application of this filter can enhance the bandwidth which is ultimately recoverable in later processing.

Delta Sigma and 24-bit Recording

- The following diagrams show the spectral analysis of the output of a DFS V (IFP) instrument compared to and ARAM 24 (delta-sigma) instrument
 - The inputs come from a high precision signal generator and contain only pure mono-frequency tones (in some cases with a low-level secondary tone)
 - Any other signals in the output are a result of instrument noise, quantization noise and harmonic distortion, all generated in the instruments.

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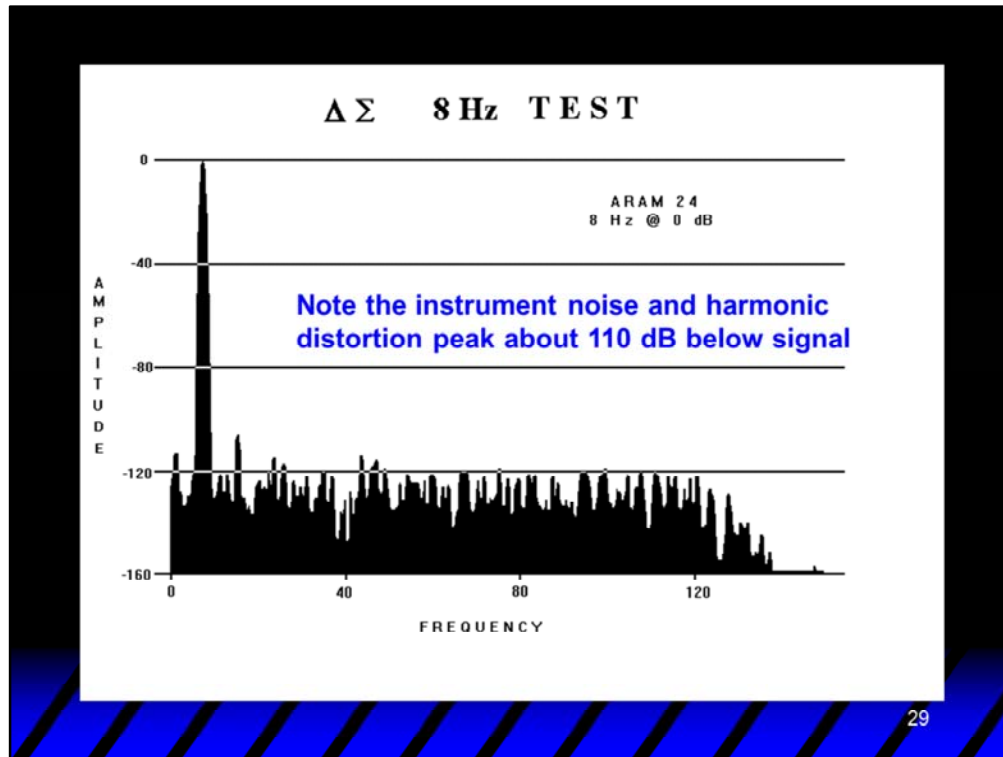
Bench tests of IFP systems compared with D-S systems provide the best appreciation for the gains made by this technology. In the following bench tests, a high precision test oscillator is used to inject one or two mono-frequency signals into the inputs of the instruments. The outputs are recorded and analyzed in the frequency domain. Any signals other than the prime input signals are due to distortions introduced by the instrumentation (harmonic distortion, electronic “hiss”, quantization noise).



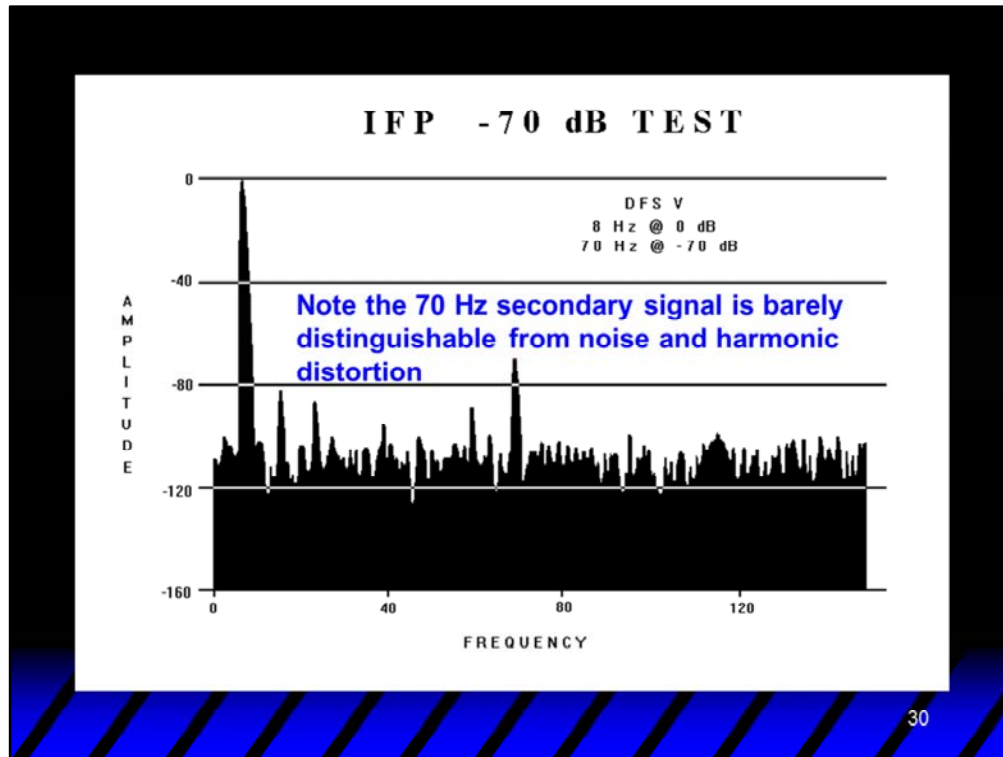
A typical IFP instrument driven by a pure 8 Hz input signal shows considerable background noise and spikes of harmonic distortion at multiples of the input frequency.

For broad band inputs, the strongest lobe of harmonic distortion would occur at all frequencies. Therefore, the strongest lobe of distortion becomes the noise floor.

Note that this system exhibits a dynamic range of just over 80 dB.

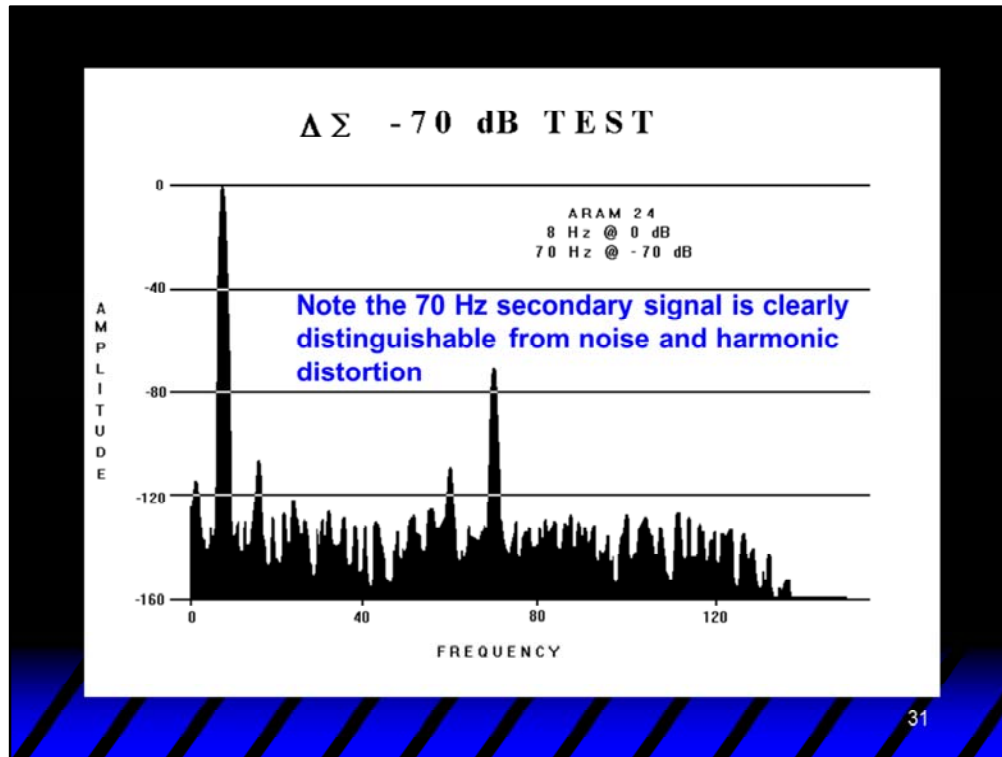


An 8 Hz pure signal injected into a D-S system exhibits a much lower noise level. This system has about 110 dB of instantaneous dynamic range.

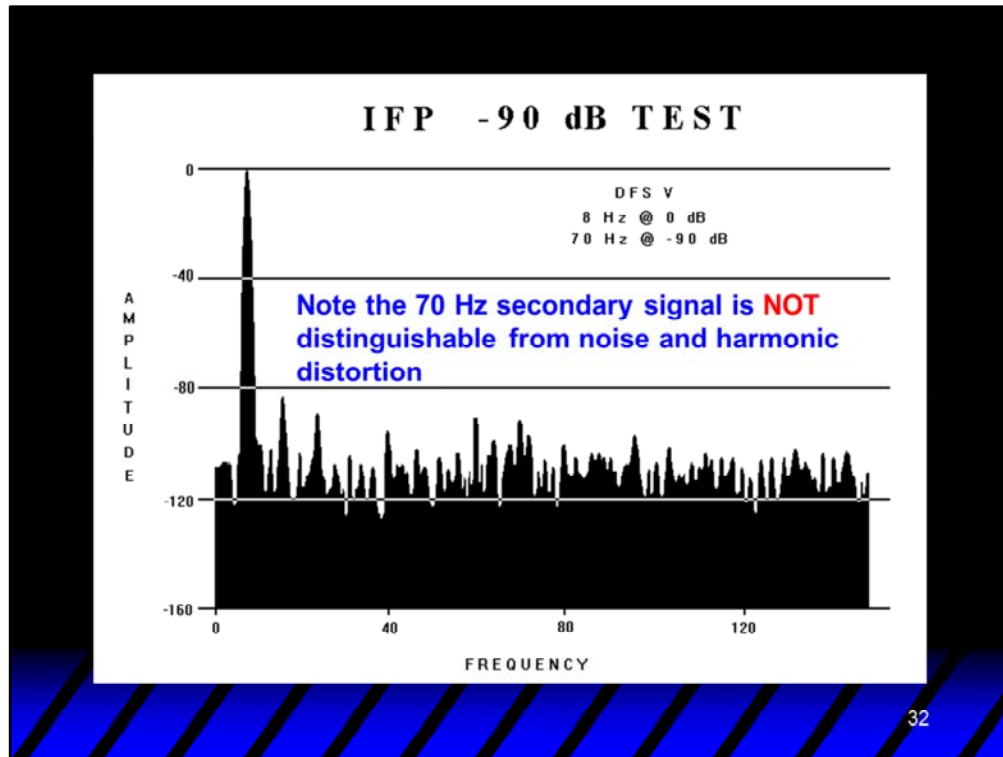


Here we have introduced a secondary input signal at 70 Hz. This signal is 70 dB weaker than the primary 8 Hz signal. It is distinguishable above the strongest harmonic distortion.

Therefore this IFP system exhibits at least 70 dB of dynamic range.

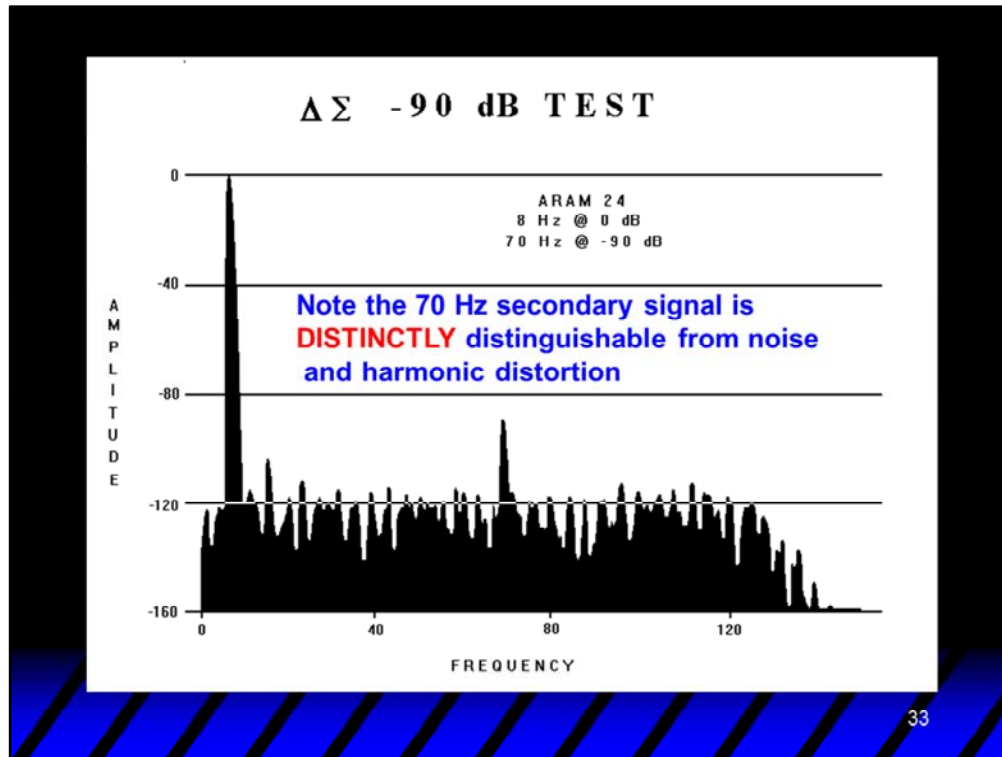


The same dual tone input injected into a D-S system reveals a much cleaner and more recognizable record of the secondary signal.



In this example, we have dropped the level of the secondary tone to 90 dB below the level of the primary tone.

This IFP system detects only one signal. The weak secondary signal is lost amongst the background noise and harmonic distortion. Obviously, this system does not have 90 dB of instantaneous dynamic range.



Even at 90 dB of separation, both primary and secondary tones are easily recognizable using this D-S recording system. This proves success of the technology. We have increased the instantaneous dynamic range of the A-D converter in our recording system.

However, we must not think that we have increased the performance of our entire seismic system by this same magnitude. We have only strengthened what used to be the weakest link in a delicate chain.

Delta Sigma and Dynamic Range

- There is a belief that the increased dynamic range of the A-D converter by adopting delta-sigma technology has increased the dynamic range of our entire seismic system by an equal degree.
- Unfortunately, delta-sigma technology has only revealed other processes which now limit our realizable dynamic range.

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In fact, the entire seismic system consists of many elements including: geophones, cables, pre-amps, A-D converter, recording tape and formats, processing algorithms, displays for interpretation and the interpreter's visualization of the processed images.

By lowering the curtain of noise imposed by our A-D converter, we can now more clearly see noise and limits imposed by some of the other elements in the seismic system.

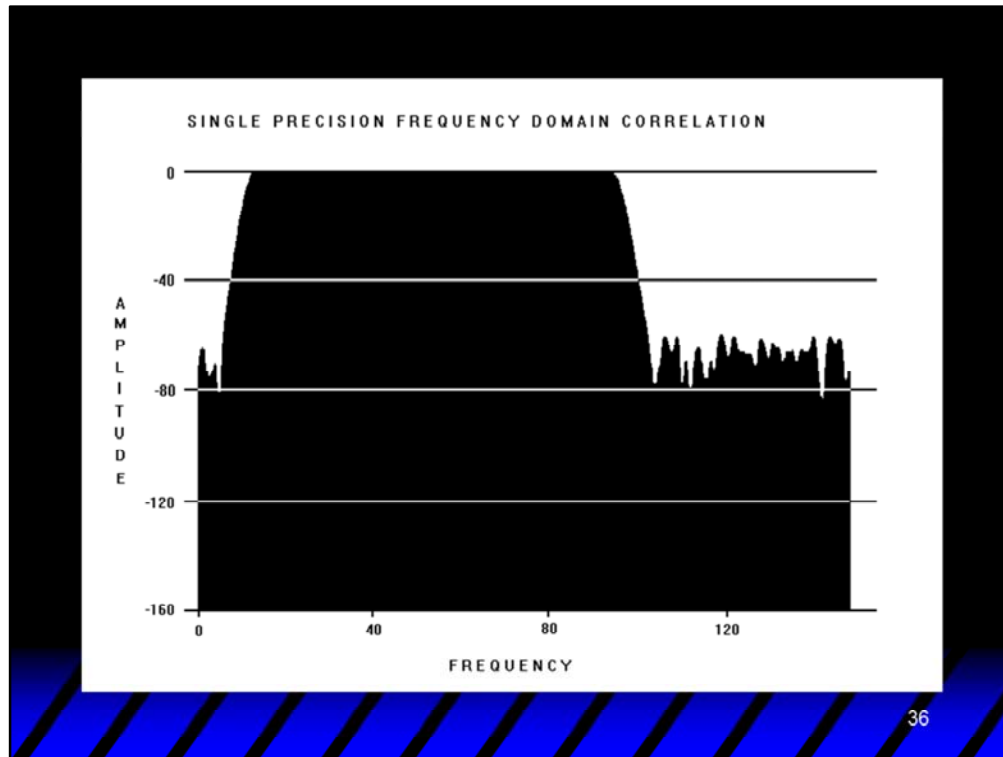
One of the most obvious is the geophones and our placement of them in the earth.

Delta Sigma and Dynamic Range

- These include field level problems such as the harmonic distortion of the pre-amplifiers, crossfeed of the cable, distortion and coupling of the geophones.
- They also include digital processing limitations.
- The following graphs show the mathematical noise introduced by frequency and time domain correlations of a pure theoretical linear sweep.

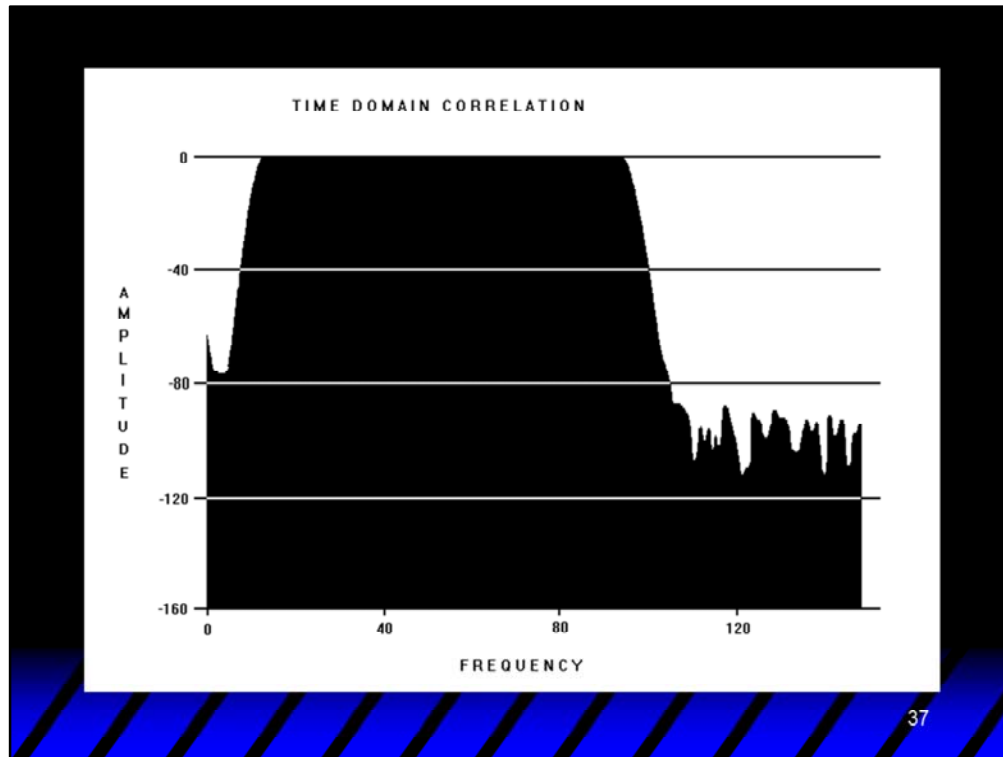
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One limit which is not commonly acknowledged is in our processing algorithms. Any computer operation (addition, multiplication, etc.) will generate a round off error after the least significant bit. This error is random and constitutes noise much like quantization noise. The accumulation of this random error over many calculations can generate a noise level which is quite noticeable in our data.



A frequency domain process involves a Fourier Transform or a Fast Fourier Transform (FFT). The number of calculations in these processes depends on the number of samples being transformed (N). A full Fourier Transform will require N^2 calculations. An FFT requires $[N \times \log(N) / 2]$ calculations. For a 1000 sample trace, this translates to a large number of calculations. Each calculation has a round off error (in the 23rd bit for single precision processors).

The above diagram is an amplitude spectrum of the result of a frequency domain correlation of a vibroseis signal showing mathematical noise at a level of 60 to 70 dB below the expected correlation.



The above diagram is an amplitude spectrum of the result of a time domain correlation of a vibroseis signal showing mathematical noise at a level of about 90 dB below the expected correlation.

This is improved over the previous picture because a time domain correlation requires only N calculations (where N is the number of input samples).

This emphasizes that for improved dynamic range, we should always try to process using time domain algorithms and at least double precision functions.

In fact, most processors employ 80-bit DSP's (digital seismic processors) for their fourier transforms. Therefore the round off error will be much lower. Nonetheless, the principles advocated here should still be adhered to because we perform many transforms and filters on our data during a typical processing flow.

Delta Sigma and Dynamic Range

- This “transform noise” is due to accumulated mathematical round off error present in any computer.
- These examples are for single precision processes.
We usually process on DSP's (digital seismic processors) which maintain 80 bits of significance (or at least in double precision).

Delta Sigma and Dynamic Range

- Nonetheless, our filtering and deconvolution processes have dynamic range which is more limited than our A-D converter.
- We must continue to employ field techniques which optimize signal to noise ratios . . .
- . . . With the condition that we do nothing to inhibit the processor's ability to recover broadband signal.

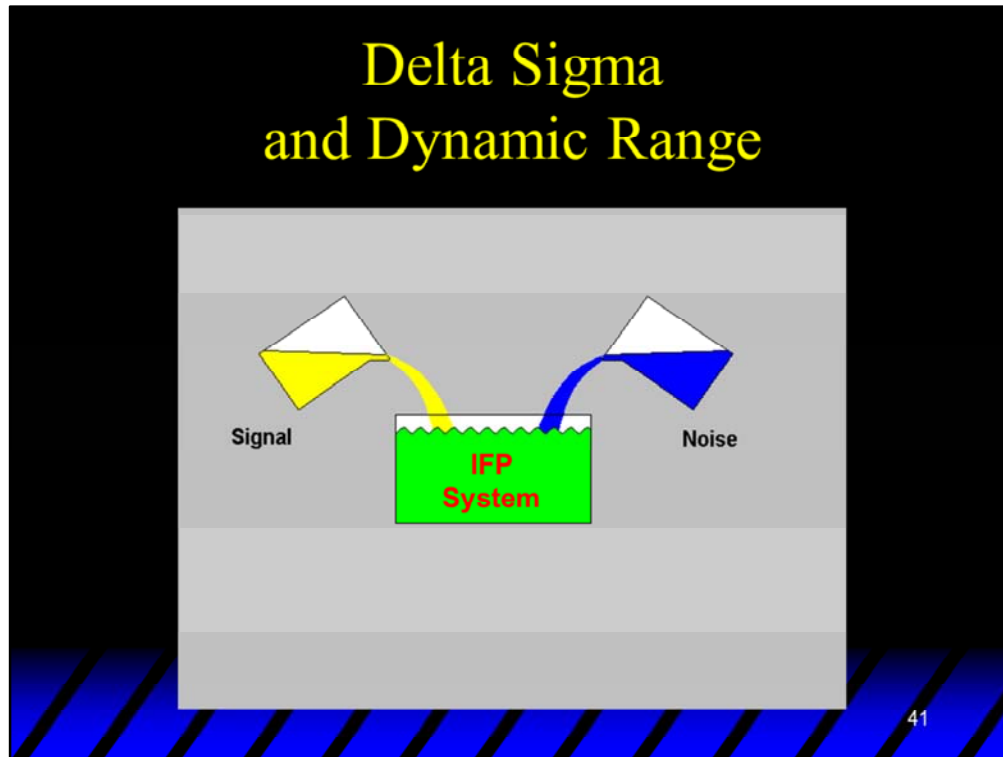
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We must not abandon sound field techniques which will deliver enhanced signal to noise ratios onto recorded tape. The cardinal rule, however, is that we must not apply methods which attenuate signal. Only methods which suppress unwanted noise where it is clearly separated from desired signal.

Delta Sigma and Dynamic Range

- The A-D converter passes bits onto tape. It is like a container which holds the only part of our field measurements that will be available to the processors.
- We must be very careful to place broad band information with optimum signal to noise ratios into this container.
- A larger container (24 bit system) is a poor excuse to muddy the waters with more noise.

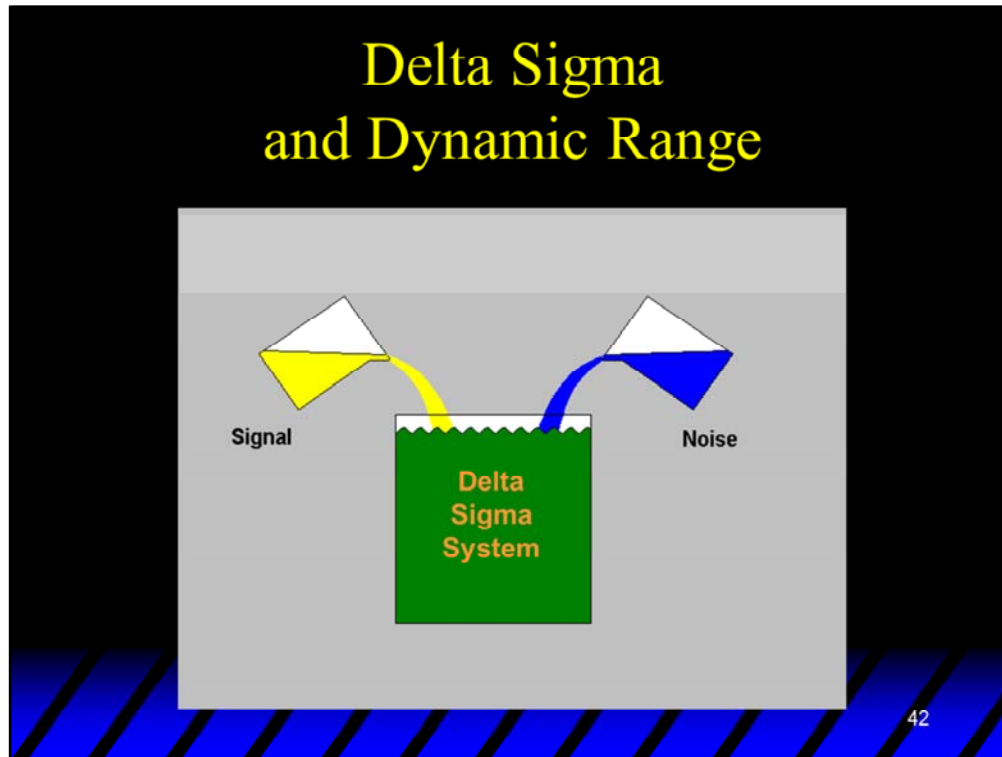
Delta Sigma and Dynamic Range



Graphically, we can think of the A-D converter as a container in which we deliver to the processor the net results of all of our field recording effort. We will always record some noise with our signal. Field techniques should be employed to deliver the best signal to noise ratio over a broad band width.

If we think of signal as a yellow fluid and noise as a blue fluid, the two will co-mingle to that the processor receives a greenish sludge from which he must extract meaningful information (yellow fluid).

An IFP system has quite limited dynamic range so we can think of the container as being quite small.



Delta Sigma systems record greater dynamic range so we can think of the container of data as being larger. However, this does not mean that we should be careless and allow more noise to enter the system. This will just result in a large container of dark green sludge from which the processor will have difficulty extracting good data.

We must still employ sound field practices and methods to enhance the signal to noise ratio which we record on tape and deliver to the processor.

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